Design of Robust PI and PID Controller for DC Motor Fuzzy Parametric Approach

M.Siva kumar¹ M. Ramalinga Raju² D.Srinivasa Rao³ and T.Bala Bhargavi⁴

^{1,3,4} Gudlavalleru Enginnering college, Department of Electrical and Electronics Engineering Gudlavalleru, AP,India ²JNTUniversity Kakinada, Dept of EEE University College of Engg, Kakinada, AP,India

ABSTRACT:

A fuzzy parametric uncertain system is an uncertain Linear Time Invariant (LTI) system with fuzzy coefficients. The design of a robust PI/PID controller for fuzzy parametric uncertain is not an easy task. In this paper, an algorithm for the design of a robust PI/PID controller is proposed. This algorithm is based on approximating the fuzzy coefficients by the nearest interval system and then a robust controller is designed using the necessary and sufficient conditions for sta-bility of the interval systems. To validate the proposed algorithm, in this paper, a robust PI/PID controller algorithm is applied to case study of DC motor fuzzy parametric uncertain system.

KEYWORDS: PI/PID controller, robust controller, fuzzy parametric uncertain system, interval system.

1 INTRODUCTION

Most of the real plant operates in a wide range of operating conditions. Robustness is then an important feature of the closed loop system. When this is the case, the controller has to be able to stabilize the plant for all operating conditions. This requires the design of a robust controller. The problem of designing a robust controller for the parametric uncertain plants, (B.M.Patre et al., 2007,2003), which have unknown but bounded parameter uncertainties attracted most of the researchers attention.

Robust control is a branch of control theory that explicitly deals with uncertainty in its approach to controller design. Robust control methods aim at achieving robust stability and/or performance in the performance of uncertainties. "Robust control refers to the control of unknown plants with unknown dynamics subject to unknown disturbances. Robust controller is a controller designed such that some level of performance of the control system is guaranteed irrespective of changes in the plant dynamics with in a pre-defined class". There are many techniques available in the literature for the design of robust controllers which are Evolutionary techniques or Numerical Optimization techniques such as Graphical methods, Adaptive control, Lyapunov method and Fuzzy control. However, there are drawbacks associated with these techniques, and they are as follows:

Numerical optimization methods are not easy to use for an operating engineer.

Graphical methods reduce complexities of plant modeling.

The Adaptive control method suffers from problems in convergence for the system parameters.

Owing to the above drawbacks, we adopt Fuzzy technology for the treatment of uncertainties. A fuzzy parametric uncertain plant is an uncertain plant in which the uncertainty can be represented using fuzzy technology and which can be approximated using an interval number. The design of a robust controller for such fuzzy uncertain parametric system is an important research problem. Several techniques are presented in (P.Husek et al., 2002, C.W.Tao et al., 2004, 2005, H.T.Nguyen et al., 1994,2000, H.K.Lam et al.,2008, C.J.Wu et al.,2009). An algorithm is proposed in (H. T. Nguyen et al., 1994) to stabilize the linear system with fuzzy representation of uncertainties. But the disadvantage associated with this algorithm is computational complexity. This disadvantage was overcome in(H. T. Nguyen et al., 2000), where the sufficient condition is derived for the crisp set to be the best approximation of the fuzzy set. An approach is proposed in(C. W. Tao,2004)to design robust controllers for uncertain systems with linguistic uncertainties approximated by fuzzy set. But the drawback involved with this approach is computational complexity in the approximation of a fuzzy set by an interval set. The linguistic information can be interpreted as a fuzzy set which is represented by a unique crisp set using interval approximation operator called the nearest interval approximation suggested in (P.Grzegorzewski et al., 2002). Using this approximation, the complexity in computations has reduced when compared to the approximation used in(C.W Tao, 2004) and gives the same corresponding interval set. For a linear system with interval structured uncertainties, a robust controller is designed using the necessary and sufficient condition in (B.M.Patre etal., 2007). The method in (B.M.Patre etal., 2007) is simple, and involves less computations and hence the robust controller can be designed easily. In this paper, an algorithm for the design of robust PI and PID controller for parametric uncertain system with a case study of DC motor speed control is proposed.

2 A BRIEF REVIEW OF INTERVAL APPROXIMATION OF UNCERTAIN SYSTEMS

An uncertain system in which the coefficients are described by fuzzy function is called an extended system or fuzzy parametric uncertain system. In case of linear systems, the fuzzy parametric uncertain system can be modeled in transfer function form as,

$$\tilde{G}(s,\tilde{p}) = \frac{b_m(\tilde{p})s^m + b_{m-1}(\tilde{p})s^{m-1} + \dots + b_0(\tilde{p})}{s^n + a_{n-1}(\tilde{p})s^{n-1} + \dots + a_0(\tilde{p})}$$
(1)

Where $\tilde{p} = (\tilde{p_1}, \tilde{p_2}, ..., \tilde{p_r}), p_{i \in X_R}$ is a vector of fuzzy numbers and X_R denotes the set of all possible fuzzy sets with real universe of discourse. If α is the degree of confidence, then the equation can be represented in interval systems as,

$$[\tilde{G}(s,\tilde{p})]\alpha = \frac{b_m(\tilde{p}\alpha)s^m + b_{m-1}(\tilde{p}\alpha)s^{m-1} + \dots + b_0(\tilde{p}\alpha)}{s^n + a_{n-1}(\tilde{p}\alpha)s^{n-1} + \dots + a_0(\tilde{p}\alpha)}$$
(2)

Where $\tilde{p\alpha} = (p_1\alpha, p_2\alpha, ..., p_r\alpha), p_{i\alpha}\epsilon[p_{i\alpha}^-, p_{i\alpha}^+]$

Where, $I = 1, 2, \dots, r$ is a vector of intervals corresponding to the α -cuts of the parameters. The membership value α can be interpreted as the degree of confidence whose value ranges from 0 to 1.

2.1 Nearest Interval Approximation:

The interval approximation operator called the nearest interval approximation is best with respect to a certain measure of distance between fuzzy numbers. Suppose A_1 and A_2 are two arbitrary fuzzy numbers with α cuts $[A_1^-(\alpha), A_2^+(\alpha)]$ and $[A_2^-(\alpha), A_2^+(\alpha)]$ respectively, the quantity $d(A_1, A_2)$ distance between the two fuzzy numbers A_1, A_2 can be given as

$$d(A_1, A_2) = \sqrt{\int_0^1 ((A_1^-(\alpha) - A_2^-(\alpha)^2) d\alpha} + \int_0^1 ((A_1^-(\alpha) - A_2^-(\alpha)^2)) d\alpha$$
(3)

After finding the solution of (3), we obtain the nearest interval approximation of A_1 as,

$$C_d(A_1) = \left[\int_{0}^{1} A_1^{-}(\alpha) d\alpha \int_{0}^{1} A_1^{+}(\alpha) d\alpha\right]$$
(4)

3 NECESSARY AND SUFFICIENT CONDITIONS FOR ROBUST STABILITY OF INTERVAL POLYNOMIALS

Consider the set of real polynomials of degree n as

$$q(s) = q_0 + q_1 s + q_2 s^2 + \dots + q_n s^n$$
(5)

The stability of such an interval polynomial represented in (5) can be determined by Kharitonovs theorem.

Kharitonov theorem. If a characteristic polynomial $\Delta(s, \tilde{d})$ with intervals is degree invariant, the system with the characteristic polynomial $\Delta(s, \tilde{d})$ is stable if and only if the systems with four kharitonov polynomials as characteristic polynomials are stable.

Let the characteristic polynomial of an uncertain system with interval parameters be

$$\Delta(s,\tilde{d}) = \sum_{i=0}^{n} d_i s^i = \sum_{i=0}^{n} [x_i, y_i] s^i$$
(6)

where $d_i = [x_i, y_i]$ According to the Kharitonovs theorem(Barmish R. et al., 1989), this interval parameter characteristic polynomial $\Delta(s, \tilde{d})$ can be represented as four fixed parameter polynomials called Kharitonov polynomials. These are:

$$\Delta^{1}(s) = x_{0} + x_{1}s + y_{2}s^{2} + y_{3}s^{3} + \dots - \dots - \dots$$
$$\Delta^{2}(s) = y_{0} + y_{1}s + x_{2}s^{2} + x_{3}s^{3} + \dots - \dots - \dots$$
$$\Delta^{3}(s) = y_{0} + x_{1}s + x_{2}s^{2} + y_{3}s^{3} + \dots - \dots - \dots$$
$$\Delta^{4}(s) = x_{0} + y_{1}s + y_{2}s^{2} + x_{3}s^{3} + \dots - \dots - \dots - \dots$$

If each and every element of the above set is a Hurwitz polynomial, then only the set of polynomial considered in(6) is stable. In (Y.Y.Nie et al., 1976), a new stability criterion for class of fixed parameter polynomials is presented. This stability criterion is extended for the determination of robust stability of interval polynomials in (B.M.Patre etal.,,2007). According to patre (B.M.Patre etal.,,2007), the robust stability of eq(5) or eq(6) can be determined by the following necessary and sufficient conditions.

Necessary conditions:

For q(s) or $\Delta(s, \tilde{d})$ to be stable, the necessary conditions to be satisfied are:

 x_i

$$y_i \ge x_i \ge 0, fori = 0, 1, 2 - - - n$$

$$x_{i+1} \ge y_{i-1} y_{i+2}, i = 1, 2, 3, - - - - n - 2$$
(7)

Sufficient conditions: For q(s) or $\Delta(s, \tilde{d})$ to be stable, the sufficient conditions to be satisfied are:

$$y_i \ge x_i > 0, i = 0, 1, 2, 3 - - - n$$

$$0.4655x_i x_{i+1} > y_{i-1} y_{i+2}, i = 1, 2, 3 - - - - n - 2$$
(8)

4 DESIGN PROCEDURE FOR THE ROBUST CONTROLLER

Consider a fuzzy parametric uncertain plant represented in its transfer function as

$$\tilde{G}(s,\tilde{u},\tilde{v}) = \frac{N(s,\tilde{u})}{D(s,\tilde{v})} = \frac{b_m(\tilde{p})s^m + b_{m-1}(\tilde{p})s^{m-1} + \dots + b_0(\tilde{p})}{s^n + a_{n-1}(\tilde{p})s^{n-1} + \dots + a_0(\tilde{p})}$$
(9)

The (9) can be represented as in interval plant by using The nearest interval approximation technique given in section 2.1. It is given as,

$$[\tilde{G}(s,\tilde{p})]\alpha = \frac{b_m(\tilde{p}\alpha)s^m + b_{m-1}(\tilde{p}\alpha)s^{m-1} + \dots + b_0(\tilde{p}\alpha)}{s^n + a_{n-1}(\tilde{p}\alpha)s^{n-1} + \dots + a_0(\tilde{p}\alpha)}$$
(10)

where

C

$$N(s, \tilde{u}) = \tilde{u}_0 + \tilde{u}_1 s + \tilde{u}_2 s^2 + \dots + \tilde{u}_m s^m$$

$$D(s, \tilde{v}) = \tilde{v}_0 + \tilde{v}_1 s + \tilde{v}_2 s^2 + \dots + \tilde{n}_n s^n$$
(11)

Where $\tilde{u}_i \epsilon[u_i^-, u_i^+]$, $\tilde{v}_i \epsilon[v_j^-, v_j^+] i = 0, 1, 2, \dots, m$ $j = 0, 1, 2, \dots, n; m \le n$ and the stabilizing PI or PID controller transfer function is given by

$$C_{PI}(s, K_1, K_2) = \frac{(k_1 s + k_2)}{s} = \frac{[K_1^-, K_1^+] + [K_2^-, K_2^+]}{s} = \frac{N_c(s)}{D_c(s)} (for PI)$$

$$PID(s, K_1, K_2, K_3) = \frac{(k_1 s + k_2 + k_3 s^2)}{s} = \frac{[K_1^-, K_1^+]s + [K_2^-, K_2^+] + [K_3^-, K_3^+]s^2)}{s} = \frac{N_c(s)}{D_c(s)} (for PID)$$

Then calculate the closed-loop polynomial of the system with PI/PID controller using

$$\Delta(s) = N(s,\tilde{u}) * N_c(s) + D(s,\tilde{v}) * D_c(s)$$
(12)

Where $N(s, \tilde{u})$ and $D(s, \tilde{v})$ are the numerator and denominator polynomials of the plant considered respectively and $N_c(s)$ and $D_c(s)$ are the numerator and denominator polynomials of PI/PID controller transfer function respectively.Now by applying the necessary and sufficient conditions of robust stability to the closed-loop polynomial derive the set of inequality constraints. Then solve the obtained inequality constraints using MATLAB-Optimization tool box(Optimization Toolbox) so as to minimize the objective function $J = \sum_{j=1}^{n} |k_j|$ and form the table with the values of controller parameters for different values of ε . Then obtain the Kharitonovs polynomials (Barmish R., 1989) to check the stability and find the closed-loop step response and verify the results.

5 DESIGN OF ROBUST CONTROLLER

CASE STUDY: Design of Robust PI Controller for DC motor: DC motor: Consider a DC motor which drives a viscously damped inertial load. The transfer function obtained between the torque and armature voltage is given by (P.S.V.Nataraj et al., 2001)

$$g(s) = \frac{T_L(s)}{V_a(s)} = \frac{K(J_l s + B_l)}{(Ls + R)(J_m s + J_l s + B_m + B_l) + K^2}$$

 $K\epsilon[0.2, 0.6], J_l\epsilon[10^{-5}, 3*10^{-5}], J_m = 2*10^{-3}, B_m = 2*10^{-5}, L = 10^{-2}H, R=1, B_l=B_m\varpi=20.$

Here, the only varying parameters are K and J_l . These parameters can be represented as fuzzy sets with triangular membership functions as $\tilde{k} = tri(0.2, 0.4, 0.6)$ and $\tilde{J}_l = tri([10]^{-5}, 2 * [10]^{-5}, 3 * [10]^{-5})$. Then the nearest interval approximation of these triangular membership functions can be obtained as:

Let \tilde{K} can be expressed as

 $\mu_k(x) = 5x - 1 for 0.2 \le x \le 0.4,$ $\mu_k(x) = 5x + 3 for 0.4 \le x \le 0.6$ $\mu_k(x) = 0 \text{ otherwise}$

Then force $\alpha \in [0,1]$ α -cuts can be obtained using method given in section 2.1.

$$[A^{-}(\alpha), A^{+}(\alpha)] = \left[\frac{\alpha+1}{5}, \frac{3-\alpha}{5}\right]$$

M Sivakuamr M Ramalinga raju D srinivasarao and T Bala Bhargavi

< 0

$$C^{-} = \int_{0}^{1} A^{-}(\alpha) d\alpha = 0.3, C^{+} = \int_{0}^{1} A^{+}(\alpha) d\alpha = 0.5$$
$$C_{d}(A) = [K^{-}, K^{+}] = [0.3, 0.5]$$

. Similarly, the nearest interval approximation of the inertial load (\tilde{j}_l) is $[J_l^-, J_l^+] = [1.5 * 10^{-5}, 2.5 * 10^{-5}]$. Now we design a controller $C_{PI}(s)$ in such a way that it stabilizes the DC motor torque in the above approximated interval. Let us consider PI controller with its transfer function defined as

$$C_{PI}(s, K_1, K_2) = \frac{\left([K_1^-, K_1^+] + [K_2^-, K_2^+]\right)}{s} = \frac{N_c(s)}{D_c(s)}$$

For the above considered system, the close-loop polynomial will be

 $\Delta(s,k,J_l,k_1,k_2) = [2.015*10^{-5}, 2.025*10^{-5}]s^3 + [0.45*10^{-5}K_1^- + 2.015*10^{-3}, 1.25*10^{-5}K_1^+ + 2.025*10^{-3}]s^2 + [0.45*10^{-5}K_2^- + 0.6*10^{-5}K_1^- + 0.09004, 1.25*10^{-5}K_2^+ + 10^{-5}K_1^+ + 0.25004]s + [0.6*10^{-5}K_2^-, 10^{-5}K_2^+].$

On applying the necessary and sufficient conditions given in section3 to this polynomial, the following inequality constraints are obtained.

$$\begin{array}{c} 0.6*10^{-5}K_2^- - 1*10^{-5}K_2^- - 1*10^{-5}K_2^+ + \varepsilon < 0 \\ 0.45*10^{-5}K_1^- + 0.6*10^{-5}K_1^- - 1.25*10^{-5}K_2^+ - 1*10^{-5}K_1^+ - 0.16 + \varepsilon < 0 \\ 0.45*10^{-5}K_1^- - 1.25*10^{-5}K_1^+ - 1*10^{-5} + \varepsilon < 0 \\ -2.7*10^{-11}K_1^{-2} - 2.025*10^{-11}K_2^-K_1^- - 9.0693*10^{-9}K_2^- - 4.172724*10^{-7}K_1^- - 1.814658*10^{-4} + \varepsilon \\ + 0.5665*10^{-11}K_2^{-2} - 0.4665*10^{-11}K_2^-K_1^- - 9.0693*10^{-9}K_2^- - 4.172724*10^{-7}K_1^- - 1.814658*10^{-4} + \varepsilon \\ + 0.5665*10^{-11}K_2^{-2} - 0.4665*10^{-11}K_2^-K_1^- - 9.0693*10^{-9}K_2^- - 4.172724*10^{-7}K_1^- - 1.814658*10^{-4} + \varepsilon \\ + 0.5665*10^{-11}K_2^-K_1^- - 9.0693*10^{-9}K_2^- - 4.172724*10^{-7}K_1^- - 1.814658*10^{-4} + \varepsilon \\ + 0.5665*10^{-11}K_2^-K_1^- - 9.0693*10^{-9}K_2^- - 4.172724*10^{-7}K_1^- - 1.814658*10^{-4} + \varepsilon \\ + 0.5665*10^{-11}K_2^-K_1^- - 9.0693*10^{-9}K_2^- - 4.172724*10^{-7}K_1^- - 1.814658*10^{-4} + \varepsilon \\ + 0.5665*10^{-11}K_2^-K_1^- - 9.0693*10^{-9}K_2^- - 4.172724*10^{-7}K_1^- - 1.814658*10^{-4} + \varepsilon \\ + 0.5665*10^{-11}K_2^-K_1^- - 9.0693*10^{-9}K_2^- - 4.172724*10^{-7}K_1^- - 1.814658*10^{-4} + \varepsilon \\ + 0.5665*10^{-11}K_2^-K_1^- - 9.0693*10^{-9}K_2^- - 4.172724*10^{-7}K_1^- - 1.814658*10^{-4} + \varepsilon \\ + 0.5665*10^{-11}K_2^-K_1^- - 9.0693*10^{-9}K_2^- - 4.07274*10^{-7}K_1^- - 1.814658*10^{-4} + \varepsilon \\ + 0.5665*10^{-11}K_2^-K_1^- - 9.0693*10^{-9}K_2^- - 4.07274*10^{-7}K_1^- - 1.814658*10^{-4} + \varepsilon \\ + 0.5665*10^{-11}K_2^-K_1^- - 9.065*10^{-11}K_2^-K_1^- - 9.065*10^{-11}K_2^- - 9.065*10^{-11}K_2^-K_1^- - 9.065*10^{-11}K_2^- - 9.065*10^{-11}K_2^- - 9.065*10^{-11}K_2^-K_1^- - 9.065*10^{-11}K_2^- - 9.05*10^{-11}K_2^-K_1^- -$$

$$\begin{split} -1.25685*10^{-11}K_1^{-2} - 9.426375*10^{-12}K_2^{-}K_1^{-} - 4.221759*10^{-9}K_2^{-} - 1.942403*10^{-7}K_1^{-} - 8.447233*10^{-5} + \varepsilon < 0 \\ -9.0675*10^{-11}K_1^{-} - 4.06103*10^{-8} + \varepsilon < 0 \\ -4.22092*10^{-11}K_1^{-} - 1.890409*10^{-8} + \varepsilon < 0 \end{split}$$

Upon solving the above constraints using the MATLAB-Optimization tool box(Optimization Toolbox) which minimizes the objective function $J = \sum_{j=1}^{n} |k_j|$, we get the controller parameter values for different ε values. For ε =0, the controller uncertain parameters obtained are

$$K_1^- = -4.9787 * 10^{-5}, K_1^+ = 9.9574 * 10^{-6} \& K_2^- = -0.01195, K_2^+ = 0.0199.$$

Then the set of Kharitonov polynomials for $K_1 = 2.48935 * 10^{-6} \& K_2 = 3.975 * 10^{-3}$ is:

$$\begin{split} &\Delta_1(s) = 2.385 * 10^{-8} + 0.09004s + 2.0254 * 10^{-3}s^2 + 2.025 * 10^{-5}s^3 \\ &\Delta_2(s) = 3.975 * 10^{-8} + 0.25004s + 2.0154 * 10^{-3}s^2 + 2.015 * 10^{-5}s^3 \\ &\Delta_3(s) = 3.975 * 10^{-8} + 0.09004s + 2.0154 * 10^{-3}s^2 + 2.025 * 10^{-5}s^3 \\ &\Delta_4(s) = 2.385 * 10^{-8} + 0.25004s + 2.0254 * 10^{-3}s^2 + 2.025 * 10^{-5}s^3 \end{split}$$

All the four Kharitonov polynomials are Hurwitz stable. Hence the designed PI controller stabilizes the DC motor system. The closed-loop step response and the frequency response for $\varepsilon = 0$ and $k_1 = 2.48935 * 10^{-6}$, $k_2 = 3.975 * 10^{-3}$ are shown in Figure 1 and Figure 2 respectively.

Design of Robust PID controller for DC motor: Now we consider the PID controller transfer function $C_P ID(s, K_1, K_2, K_3)$ as

$$C_{PID}(s, K_1, K_2, K_3) = \frac{([K_1^-, K_1^+]s + [K_2^-, K_2^+] + [K_3^-, K_3^+]s^2)}{s}$$

The closed-loop polynomial after calculation will be

$$\begin{aligned} \Delta(s,k,J_l,k_1,k_2,k_3) &= [0.45*10^{-5}K_3^- + 2.015*10^{-5}, 1.25*10^{-5}K_3^+ + 2.025*10^{-5})s^3 + \\ &= [0.45*10^5K_1^- + 0.6*10^{-5}K_3^- + 2.0154*10^{-3}, 1.25*10^{-5}K_1^+ + 10^{-5}K_3^+ + 2.0254*10^{-3}]s^2 + \\ &+ [0.45*10^{-5}K_2^- + 0.6*10^{-5}K_1^- 0.09004, 1.25*10^{-5}K_2^+ + 10^{-5}K_1^+ + 0.25004]s + [0.6*10^{-5}K_2^-, 10^{-5}K_2^+] \end{aligned}$$

On applying the necessary and sufficient conditions in section3, we obtain a set of inequality constraints. Hence the optimization problem is to find k_1, k_2, k_3 such that the objective function $\sum_{j=1}^3 |k_j|$ is minimized subject to the inequalities constraints. Upon solving the inequality constraints, we get the controller parameters for $\varepsilon = 0$ as $K_1^- = -4.9787 * 10^{-6}, K_1^+ = 9.9574 * 10^{-6} \& K_2^- = -0.010256, K_2^+ = 0.01709 \& K_3^- = 3.634456 * 10^{-4}, K_3^+ =$ $1.010678 * 10^{-3}$. Using these parameters, the Kharitonov polynomials for closed loop system are formulated. It has been observed that the four Kharitonov polynomials are Hurwitz stable. The closed-loop step response and frequency responses are shown in Figure 3 and Figure 4 respectively

By using the proposed method, the system reaches its steady state value quickly when compared to the design methodology used in (R.J.Bhiwani etal.,2011). From Figure 1 and Figure 2 it has been observed that the ripple contents in torque of DC motor are minimized with the proposed PI/PID controller. Hence there will be no jerks in the operation of dc motor. So the DC motor drives a viscously damped inertial load very smoothly. Hence the performance of motor is improved by reducing the torque ripples using proposed robust PI/PID controller.



Figure 1: Step response of the Closed-loop system of the DC motor with PI Controller

Figure 2: Frequency response of the Closed-loop system of the DC motor with PI Controller

Bode Diagram





Figure 3: Step response of the closed-loop system with PID Controller

Figure 4: Frequency response of the closed-loop system with PID Controller.

Bode Diagram



6 CONCLUSION

A robust PI and PID controller algorithm is proposed in this paper. The proposed algorithm robustly improves stability of the plant in a given interval. To show the efficacy of the proposed algorithm, a DC motor transfer function obtained between load torque versus armature voltage with varying parameters of K (armature gain) and inertial load J_L is considered. The performance of DC Motor is improved by reducing the torque ripples by design the robust PI / PID Controller. This example is simulated through MATLAB-Optimization tool box and convincing results are obtained. It has been observed from the simulation results that the designed PI/PID controller stabilizes the plant.

REFERENCES

- [1] B.M.Patre and P.J.Deore, *Robust stability and performance for interval process plants*, ISA Transactions,vol.46(2007),343-349.
- [2] B. M.Patre and P. J. Deore, *Robust stabilization of interval plants*, Europian Control Conference ECC-03, University of Cambridge, Sept. 2003.
- [3] R.J.Bhiwani and B.M.Patre, *Design of Robust PI/PID Controller for Fuzzy Parametric Uncertain Systems*, International Journal of Fuzzy systems, vol.13, No.1, March 2011.
- [4] P. Husek and R. Pytelkova, Analysis of systems with parametric uncertainty described by fuzzy functions, Proc. Of 10thMediterranean Conference on Control and Automation MED2002, Lisbon, Portugal, 9-12 July, 2002.
- [5] C. W. Tao, Robust control of fuzzy systems with fuzzy representation of uncertainties, Soft Computing, vol. 8, pp.163-172, Springer-Verlag, 2004.
- [6] C. W. Tao and J. S. Taur, Robust fuzzy control for a plant with fuzzy linear model, IEEE Transaction on Fuzzy Systems, vol. 13, pp.30-41, Feb. 2005.
- [7] H. T. Nguyen and V. Kreinovich, *How stable is a fuzzy linear system*, Proc. of third IEEE Conference on Fuzzy Systems, pp.1023-1027, June 1994.
- [8] H. K. Lam and B. W. K. Ling, Computational effective stability conditions for time-delay fuzzy sys-tems, International Journal of Fuzzy Systems, vol. 10, no. 1, pp. 61-70, March 2008.
- [9] C. J. Wu, C. N. Ka, Y. Fu, and C. H. Tseng, A genetic based design of auto tuning fuzzy PID controllers, International Journal of Fuzzy Systems, vol. 11, no. 1, pp. 49-58, March 2009.
- [10] H. T. Nguyen, W. Pedrycz, and V. Kreinovich, On approximation of fuzzy sets by crisp sets: from continuous controloriented defuzzification to discrete decision-making, Proc. of the first Internation-al Conference on Intelligent Technologies, Thailand, pp. 254-260, 2000.
- [11] P. Grzegorzewski, Nearest interval approximation of a fuzzy number, Fuzzy Sets and Systems, vol. 130, no. 3, pp. 321-330, 2002.
- [12] P.S.V.Nataraj, S. Sheela, and A.K. Prakash INTERVAL QFT: A Mathematical and Computational Enhancement Of QFT Proc.5th Int.Conf On QFT and Frequency Domain me-thods, Pampolna. Spain, August 1, 2001.
- [13] Optimization Toolbox: Mathworks Inc. USA. (www.mathworks.com)
- [14] Barmish R., 1989, A generalization of K-four polynomial concept for Robust stability problems with linearly dependent coefficient perturbations, IEEE Trans.on Automatic Control, vol.34, no.2.
- [15] Y.Y.Nie., A new class of criterion for the stability of the polynomial, Acta Mechanicia Sini-ca,pp.110-116,1976.